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Discussion of Complement Test to Assess Ratio Error

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The compliment test is used to assess an instruments ability to provide accurate ratio measurements using two resistors assuming no a priori knowledge of their values. It is assumed the resistors are stable over the measurement period and not subject to other significant variation over the measurement period.

The measurement is made by recording the ratio of one resistor to another and the measurement after the resistors are switched. One set of measurements is made with R_A on terminal 1 and R_B on terminal 2. The second measurement is made with R_A on terminal 2 and R_B on terminal one. This can be referred to as a measurement and the reciprocal. The product of the reciprocal should be unity.

$$(R_A/R_B)*(R_B/R_A)=1 \qquad [1]$$

It can also be said that this is the value for two measurements, one set with R_A on terminal 1 and one set on terminal 2. The quotients are dimensionless ratios and so the error will be in ratio or PPM for the equation 2.

$$1-[(R_A/R_B)*(R_B/R_A)]*10^6/2=\text{Average Ratio Error } (\bar{E} \text{ ppm})\ 10^6 \qquad [2]$$

Equation 2 produces the average ratio error for a specific set resistors but it does not identify if there is a bias for terminal 1 versus terminal 2 as R_S.

$$R_A/R_B=A \ \&\ R_B/R_A=B \qquad [3]$$

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The difficulty here arises when one tries to determine the error for two situations A & B, which represent the forward and reverse measurement in relation to the resistor terminal.

One additional condition must be met to prove the bridge is within specification. The result of equation 2 must be at least one half of the specified tolerance in PPM.

$$\bar{E}\ \text{ppm} = 1 - [(A*B)/2]*10^6 \qquad [4]$$

Examples: Resistor R_1=100.0045600 & Resistor R_2=99.9970000

R_1=100.00456 Ohms
R_2=99.997000 Ohms

The resulting values are dimensionless ratios:

$$A = R_1/R_2 = 1.000075602 \qquad [5]$$

$$B = R_2/R_1 = 0.999924403 \qquad [6]$$

$$\bar{E} = \left[1 - \frac{A*B}{2}\right] * 10^6 = 0 \quad \text{ppm} \qquad [7]$$

So we would really have A and B plus some error associated with the system if the values of R1 and R2 are true.

$$A + A_{Error} = 1.000075658 \qquad [8]$$

$$B + B_{Error} = 0.999924438 \qquad [9]$$

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If $A_{Error} = 0.000000056$ & $B_{Error} = 0.000000035$

$$\left[1 - \frac{A + A_{Error} * B + B_{Error}}{2}\right] * 10^6 = 0.045\,ppm \qquad [10]$$

From Equation 10 it can readily be seen that from a mathematical point of view the resulting value from the compliments check is simply the average value for two readings or conditions. Although it is unlikely, there is a possibility that condition A could be one value and condition B could be zero. The compliments equation results in an average value thus masking the imbalance. In an extreme case one condition could be greater than the overall specification but when the two conditions are averaged a specification within tolerance is achieved.

The simplest solution to the problem is to not divide by two but use the total error from both measurements. By not averaging one is assured the ratio error is in fact below the instruments specification.

The equation would be:

$$1-[(R_1/R_2)*(R_2/R_2) = E \qquad [11]$$

Ideally the error value produced from equation 11 should be at least half of the ratio specification. This would ensure that any single condition could only be a maximum of one half of the instrument specification.

Given by the following:

$$1-[(R_1/R_2)*(R_2/R_2) \leq \tfrac{1}{2}(I_{spec}) \qquad [12]$$

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Where I_{Spec} = the Instrument Ratio Specification in PPM

Discussion:
A limit could be placed on the result to assure the instrument is within specification. It is also very unlikely that that the extreme condition discussed would be seen in a properly operating Dc or AC bridge. However if a compliments test did not meet the conditions of equation 12 then further investigation into its condition would be warranted. This discussion is meant to clarify the mathematical equation used with the compliments test; it does not change in any way how the test is performed.

The value obtained is a ratio error in ppm and should be converted to temperature using the appropriate relationships. The next section explores larger ratios, the discussion is limited to ratios suitable to AC bridges used in thermometry but the principles can easily be applied to larger ratios and DC measurements. (more to come)